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# 1. MEASURES

## 1.1 Direct and indirect measures

In the measurement of physical quantities, both direct and indirect methods are used.

1. **Direct measurement** refers to directly measuring the quantity for which the experiment is performed.
2. **Indirect measurement** involves first measuring some primary quantities and then using them to calculate the required quantity.

Examples of direct measurement include:

- Mass: Measured using a balance or scale.
- Distance: Determined using rulers, tape measures, or distance-measuring instruments like a laser range-finder.
- Time: Measured using clocks or timers.

Examples of indirect measurement include:

- Wind speed: Calculated using measurements of pressure gradients, Doppler radar, or the rotational speed of devices like cup anemometers.
- Area: Indirectly determined by measuring length and width and then multiplying these dimensions together.

■ **Example 1.1** Determine whether each of the following quantities is measured directly or indirectly.

speed, area, volume, density, length

**Answer:**

speed: indirectly, area: indirectly, volume: indirectly, density: indirectly, length: directly. ■

**Exercise 1.1** Which of the following quantities is measured indirectly?

- A) mass
- B) time
- C) volume
- D) length
- E) none

**Exercise 1.2** You measure the diameter of a ball with a ruler and then calculate its volume using a formula. This is a:

- A) Indirect measurement
- B) Direct measurement
- C) Estimated measurement
- D) Relative measurement
- E) Qualitative measurement

**Exercise 1.3** A scientist wants to determine the density of a rock sample. Which of the following methods is an indirect measurement?

- A) Measuring the mass of the rock sample on a scale.
- B) Measuring the volume of the rock sample using a graduated cylinder.
- C) Dividing the mass of the rock sample by its volume.
- D) Measuring the length, width, and height of the rock sample.
- E) None of them.

## 1.2 Fundamental and derived quantities

Physical quantities are often divided into fundamental quantities and derived quantities.

1. **Fundamental quantities** are those which are independent of other physical quantities.
2. **Derived quantities** are those which are derived from fundamental quantities.

Examples of Fundamental quantities include:

- length
- mass
- time
- electric current

- luminous intensity
- amount of substance
- thermodynamic temperature

Examples of Fundamental quantities include:

- area
- volume
- density
- speed
- acceleration
- force
- velocity

■ **Example 1.2** An engineer is designing a new high-speed train. They need to calculate the force required to accelerate the train to a specific speed within a set timeframe. Determine whether each of the following quantities is fundamental or derived.

Train mass, Target speed, Acceleration, Force required

**Answer:**

Train mass: fundamental, Target speed: derived, Acceleration: derived, Force required: derived. ■

**Exercise 1.4** A scientist is studying the cooling rate of a hot cup of coffee. They measure the coffee's temperature at different time intervals. Which of the following are fundamental?

- A) Coffee temperature
- B) Rate of temperature change
- C) Heat loss from the coffee
- D) Cooling constant of the coffee cup
- E) None of them

**Exercise 1.5** Definition of derived quantities:

- A) Physical quantity that cannot be derived from other physical quantities.
- B) Physical quantities that are derived from the combinations of base quantities through multiplication or division or both these operations.
- C) Physical quantity with magnitude only.
- D) Physical quantity that has both magnitude and direction.
- E) None of them

**Exercise 1.6** You measure the length of a book with a ruler. This is a:

- A) Derived quantity
- B) Fundamental quantity
- C) Dependent quantity
- D) Relative quantity
- E) Variable quantity

### 1.3 SI units of measurement

A number alone is not sufficient to describe a physical quantity. The unit defines the magnitude of a measurement. For example, when measuring length, the unit used could be a foot or a meter, each of which describes a different magnitude of length. The International System of Units (SI) is the modern form of the metric system and comprises seven base units, using twenty metric prefixes to denote decimal multiples or sub-multiples of these base units.

**SI base units:**

- **meter** (symbol: m) unit of length
- **kilogram** (symbol: kg) unit of mass
- **second** (symbol: s) unit of time
- **ampere** (symbol: A) unit of electric current
- **kelvin** (symbol: K) unit of temperature
- **candela** (symbol: cd) unit of luminous intensity
- **mole** (symbol: mol) unit of amount of substance

In addition, SI also includes units of measurement derived from the seven SI base units. These derived units are combinations of two or more of the base units. We derive units for these quantities by using their definitions and algebraic relationships linking different quantities. By substituting the base units into the formula, we get derived units.

■ **Example 1.3 Unit of velocity:**

$$v = \frac{\Delta x}{\Delta t} \rightarrow [v] = \frac{m}{s} \quad (1.1)$$

■ **Example 1.4 Unit of acceleration:**

$$a = \frac{\Delta v}{\Delta t} \rightarrow [a] = \frac{m}{s \cdot s} = \frac{m}{s^2} \quad (1.2)$$

■ **Example 1.5 Unit of force:**

$$F = ma \rightarrow [F] = \frac{kg \cdot m}{s^2} = N \quad (1.3)$$

■ **Example 1.6 Unit of energy:**

$$E = \frac{1}{2}mv^2 \rightarrow [E] = \frac{kg \cdot m^2}{s^2} = J \quad (1.4)$$



■ **Example 1.7 Unit of power:**

$$P = \frac{E}{\Delta t} \rightarrow [P] = \frac{kg \cdot m^2}{s^3} = W \quad (1.5)$$

■

■ **Example 1.8 Unit of pressure:**

$$P = \frac{F}{A} \rightarrow [P] = \frac{kg}{m \cdot s^2} = Pa \quad (1.6)$$

■

■ **Example 1.9 Unit of momentum:**

$$p = mv \rightarrow [p] = \frac{kg \cdot m}{s} \quad (1.7)$$

■

■ **Example 1.10 Unit of density:**

$$\rho = \frac{m}{V} \rightarrow [\rho] = \frac{kg}{m^3} \quad (1.8)$$

■

**Exercise 1.7** Which of the following quantities has the same unit as energy?

- A) Power
- B) Work
- C) Force
- D) Momentum
- E) None

■

**Exercise 1.8** Which of the following is a fundamental unit?

- A)  $kg \cdot m^{-3}$
- B)  $m^3$
- C)  $N \cdot m$
- D)  $kg$
- E)  $kg \cdot cm$

■

**Exercise 1.9** Which of the units of the following physical quantities is not derived unit?

- A) Area
- B) Pressure
- C) Mass
- D) Density
- E) None of them

## 1.4 Metric system and CGS System

Measurements play an important role in physics. A unit has to be defined before any kind of measurement can be made, as it provides a standard for comparing physical quantities.

Different systems of units have been used in the past, each tailored to specific needs and conventions:

- The centimetre-gram-second (**CGS**) or Gaussian system, which is often used in scientific research, particularly in electromagnetism.
- The metre-kilogram-second (**MKS**) or Metric system, which forms the basis for the modern SI units and is widely used in most countries for everyday measurements.
- The foot-pound-second (**FPS**) or British engineering system, primarily used in the United States for engineering and construction.

The new system, which has now gained universal acceptance, is the International System of Units, usually called SI units. This system is designed to be logical and coherent, with a single set of base units from which all other units can be derived. It ensures consistency and standardization across scientific and engineering disciplines worldwide, facilitating better communication and collaboration.

**Exercise 1.10** The CGS system uses centimeters for length, grams for mass, and seconds for time. What unit would you use to measure the density of a rock in this system?

- A) Newton per square centimeter
- B) Gram
- C) Second
- D) Grams per cubic centimeter
- E) None of the above

**Exercise 1.11** The CGS system uses centimeters for length, grams for mass, and seconds for time. What unit would you use to measure the volume of a liquid in this system?

- A) Cubic centimeter
- B) Gram
- C) Second
- D) Grams per cubic centimeter
- E) None of the above

## 1.5 Physical dimensions of quantities

The dimension of a physical quantity describes how it relates to the fundamental quantities of mass, length, and time. The dimension of unit mass is denoted by **M**, for unit length by **L**, and for unit time by **T**. Understanding dimensions is crucial because it helps in verifying the correctness of equations and ensuring consistency in physical formulas.

■ **Example 1.11** Let Velocity is the rate of change of displacement with time.

$$v = \frac{\Delta x}{\Delta t} \rightarrow \frac{\text{dimension of displacement}}{\text{dimension of time}} = \frac{L}{T} \quad (1.9)$$

Here, the dimension of velocity is derived by considering the displacement (**L**) over time (**T**). ■

■ **Example 1.12** Density is mass per unit volume.

$$\rho = \frac{m}{V} \rightarrow \frac{\text{dimension of mass}}{\text{dimension of volume}} = \frac{M}{L^3} \quad (1.10)$$

In this case, the dimension of density is found by dividing mass (**M**) by volume (**L**<sup>3</sup>), showing how mass is distributed in a given volume. ■

Dimensions play a fundamental role in dimensional analysis, which is a technique used to convert one set of units to another, check the consistency of equations, and derive relationships between different physical quantities. Dimensional analysis is a powerful tool because it relies solely on the units and dimensions, without needing detailed knowledge of the physical processes involved. Below is a table of a few important physical quantities and their dimensions:

Table 1.1: Important physical quantities and their dimensions

Physical Quantity	Units	Dimensions
Velocity	$m.s^{-1}$	$L.T^{-1}$
Acceleration	$m.s^{-2}$	$L.T^{-2}$
Force	$N$	$M.L.T^{-2}$
Momentum	$kg.m.s^{-1}$	$M.L.T^{-1}$
Density	$kg.m^{-3}$	$M.L^{-3}$
Pressure	$N.m^{-2}$	$M.L^{-1}.T^{-2}$

**Exercise 1.12** Work, defined as force multiplied by displacement, tells us how much energy is transferred. What is the dimension of work?

- A)  $M.L$
- B)  $M.L^2$
- C)  $M.L.T$
- D)  $M.L^2.T^{-2}$
- E) Dimensionless

**Exercise 1.13** The dimension of pressure is given as  $M^x L^y T^z$ , deduce the values of x, y, and z. (Hint: Pressure= Force/Area, the unit is  $N.m^{-2}$ )

- A)  $x=1, L=1, T=2$
- B)  $x=1, L=1, T=-2$
- C)  $x=1, L=-1, T=2$
- D)  $x=1, L=-1, T=-2$
- E) None of them

**Exercise 1.14** The frequency  $f$  of oscillation of a mass  $m$  attached to a spring of spring constant  $k$  is related to  $m$  and  $k$ . Find the relationship between  $f$ ,  $m$  and  $k$ . (Hint:  $[f] = s^{-1}, [k] = N/m$ )

- A)  $f = \frac{m}{k}$
- B)  $f = \sqrt{\frac{k}{m}}$
- C)  $f = \sqrt{\frac{m}{k}}$
- D)  $f = \sqrt{\frac{m}{k^2}}$
- E) None of them

## 1.6 Multiples and sub-multiples

Multiples are factors used to create larger forms of SI units, whereas sub-multiples are factors used to create smaller forms of SI units. For example, a centimeter is a sub-multiple and a kilometer is a multiple of a meter. Using multiples and sub-multiples allows us to work with quantities that are more convenient in size for practical measurements, avoiding very large or very small numbers.

The International System of Units (SI) comprises seven base units that use twenty metric prefixes to denote decimal multiples or sub-multiples of the base unit. These prefixes help to standardize

measurements across different scales, making it easier to communicate and understand the magnitude of various physical quantities.

Table 1.2: Multiples and sub-multiples

Power	Prefix	Symbol	Power	Prefix	Symbol
$10^{-18}$	atto-	A	$10^1$	deka-	da
$10^{-15}$	femto-	F	$10^2$	hekto-	h
$10^{-12}$	pico-	p	$10^3$	kilo-	k
$10^{-9}$	nano-	n	$10^6$	mega-	M
$10^{-6}$	micro-	$\mu$	$10^9$	giga-	G
$10^{-3}$	milli-	m	$10^{12}$	tera-	T
$10^{-2}$	centi-	c	$10^{15}$	peta-	P
$10^{-1}$	deci-	d	$10^{18}$	exa-	E

Scientific notation is a method of expressing numbers in the form  $a \times 10^b$ , where  $a$  is a decimal such that  $1 \leq a < 10$  and  $b$  is a positive or negative integer. This notation is particularly useful for representing very large or very small numbers in a compact and precise manner. The number of digits in  $a$  represents the number of significant figures in the number.

Using scientific notation helps simplify calculations and comparisons, especially when dealing with extreme values. It reduces the complexity of arithmetic operations and provides a standardized way to communicate measurements across different scientific disciplines.

In addition to its practical use in simplifying calculations, scientific notation is essential for maintaining consistency and clarity in scientific communication. It allows scientists to easily read, write, and understand extremely large or small numbers without ambiguity.

■ **Example 1.13** In the table below, the numbers are written in scientific notation and the number of significant figures is specified. ■

Table 1.3: Scientific notation and number of significant figures

Number	Scientific notation	Number of s.f.
608	$6.08 \times 10^2$	3
206000	$2.06 \times 10^5$	3
400	$4 \times 10^2$	1
0.000309	$3.09 \times 10^{-4}$	3
0.005600	$5.600 \times 10^{-3}$	4

Unit conversion is a multi-step process that involves multiplication or division by a numerical factor, selecting the correct number of significant digits, and rounding as needed. This process ensures that measurements are expressed in appropriate and consistent units, facilitating clear communication and accurate calculations.

Conversions are essential in physics and engineering because different systems of measurement are used around the world, and scientific data often needs to be compared and analyzed in a common unit system. Unit conversion allows for consistency and precision in these comparisons.

■ **Example 1.14** To convert a speed from kilometers per hour (km/h) to meters per second (m/s):

$$36 \frac{\text{km}}{\text{h}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 10 \frac{\text{m}}{\text{s}} \quad (1.11)$$

In this conversion:

- Multiply by  $10^3 \text{ m}/1 \text{ km}$  to convert kilometers to meters.
- Multiply by  $1 \text{ h}/3600 \text{ s}$  to convert hours to seconds.
- The result is  $10 \frac{\text{m}}{\text{s}}$ .

This shows how a commonly used speed unit in everyday life, km/h, is converted to a unit more suitable for scientific calculations, m/s, highlighting the importance of proper unit conversion for different contexts. ■

■ **Example 1.15** To convert an area rate from square meters per nanosecond ( $\frac{\text{m}^2}{\text{ns}}$ ) to square centimeters per millisecond ( $\frac{\text{cm}^2}{\text{ms}}$ ):

$$36 \frac{\text{m}^2}{\text{ns}} \times \frac{1 \text{ cm}^2}{(10^{-2})^2 \text{ m}^2} \times \frac{10^{-3} \text{ ns}}{10^{-9} \text{ ms}} = 36 \times 10^{10} \frac{\text{cm}^2}{\text{ms}} \quad (1.12)$$

In this conversion:

- Multiply by  $1 \text{ cm}^2/(10^{-2})^2 \text{ m}^2$  to convert square meters to square centimeters.
- Multiply by  $10^{-3} \text{ ns}/10^{-9} \text{ ms}$  to convert nanoseconds to milliseconds.
- The result is  $36 \times 10^{10} \frac{\text{cm}^2}{\text{ms}}$ .

This example illustrates the complexity that can arise in unit conversions, especially when dealing with very small or very large units. Converting between such units requires careful consideration of the conversion factors and an understanding of the relationships between different units. ■

In summary, mastering unit conversion is crucial for accurate scientific work and communication. It allows scientists and engineers to compare results, replicate experiments, and apply findings universally, regardless of the measurement system initially used.

**Exercise 1.15** What is 36 mph (miles per hour) in m/s (meters per second)? (1 mile = 1609 m)

- A) 1.609 m/s
- B) 16.09 m/s
- C) 1609 m/s
- D) 160.9 m/s
- E) 10 m/s

**Exercise 1.16** The number 300,000,000 can be written as

- A)  $3 \times 10^6$
- B)  $3 \times 10^7$
- C)  $3 \times 10^8$
- D)  $3 \times 10^9$
- E)  $3 \times 10^{-9}$

**Exercise 1.17** How many nanometers are in a kilometer?

- A)  $10^9$
- B)  $10^{-9}$
- C)  $10^{-12}$
- D)  $10^{12}$
- E)  $10^{27}$

## 1.7 Vectors and vector operations

Quantities in physics are categorized as either scalars or vectors. Scalars are quantities that have only magnitude, such as distance, speed, mass, time, density, electric potential, electric charge, gravitational potential, temperature, volume, work, energy, and power. Vectors, on the other hand, possess both magnitude and direction. Examples of vectors include displacement, velocity, acceleration, force, weight, electric field, magnetic field, gravitational field, momentum, area, and angular velocity.

A vector is typically represented by a straight arrow. The direction of the arrow indicates the direction of the vector, while the length of the arrow corresponds to the vector's magnitude. Two vectors are considered equal if both their magnitudes and directions are identical.

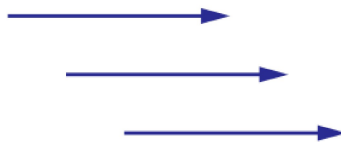


Figure 1.1: Vectors

### Addition of Two Vectors

Vector addition involves combining two vectors to produce a resultant vector. This can be done graphically using the head-to-tail method or analytically using their components.

### Subtraction of Two Vectors

Vector subtraction is similar to addition but involves reversing the direction of the vector being subtracted before combining it with the other vector.

### Multiplication of a Vector by a Scalar

A vector can be multiplied by a scalar (a real number). When a vector  $\vec{a}$  is multiplied by the scalar

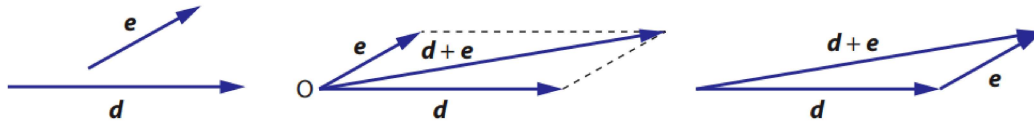


Figure 1.2: Addition of Two Vectors

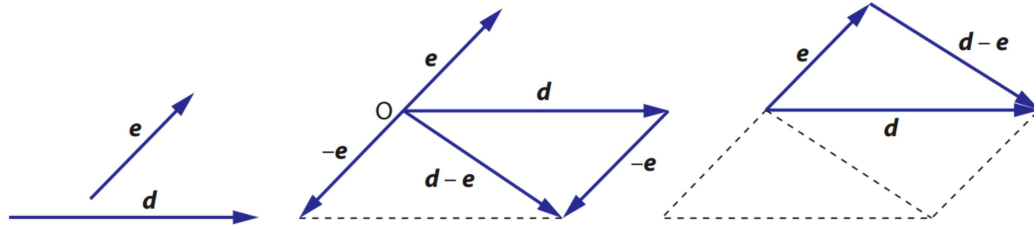


Figure 1.3: Subtraction of Two Vectors

2, the resulting vector is in the same direction as  $\vec{a}$  but twice as long. If  $\vec{a}$  is multiplied by  $-0.5$ , the resulting vector is in the opposite direction to  $\vec{a}$  and half as long.

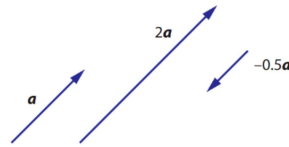


Figure 1.4: Multiplication of a Vector by a Scalar

### Product of Two Vectors

There are two types of vector products: the dot product and the cross product.

#### Dot Product

The dot product (or scalar product) of two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes  $|\vec{a}|$  and  $|\vec{b}|$  respectively is given by:

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \quad (1.13)$$

where  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ . The result is a scalar. For vectors expressed in component form:

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \quad \vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k} \quad (1.14)$$

their dot product is:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad (1.15)$$

■ **Example 1.16** Calculate the dot product of the vectors  $\vec{a} = 3\vec{i} - 4\vec{j}$ ,  $\vec{b} = 2\vec{i} + 5\vec{j}$ .

**Answer:**

$$\vec{a} \cdot \vec{b} = 3 \times 2 + (-4) \times 5 = 6 - 20 = -14$$

■



■ **Example 1.17** Find the dot product of two vectors having magnitudes of 4 units and 5 units, and the angle between the vectors is  $60^\circ$ .

**Answer:**

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = 4 \times 5 \times \cos 60 = 10$$

### Cross Product

The cross product (or vector product) of two vectors  $\vec{a}$  and  $\vec{b}$  is given by:

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n} \quad (1.16)$$

where  $\theta$  is the angle between the vectors, and  $\hat{n}$  is a unit vector perpendicular to the plane containing  $\vec{a}$  and  $\vec{b}$ . For vectors expressed in component form:

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \quad \vec{b} = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k} \quad (1.17)$$

their cross product can be expanded using distributivity, and the result is a new vector that is perpendicular to both  $\vec{a}$  and  $\vec{b}$ .

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2) \vec{i} - (a_1 b_3 - a_3 b_1) \vec{j} + (a_1 b_2 - a_2 b_1) \vec{k} \quad (1.18)$$

These fundamental vector operations are crucial in the study of kinematics, as they provide the tools needed to analyze motion in a multidimensional context.

■ **Example 1.18** Calculate the cross product of the vectors  $\vec{a} = 1\vec{i} + 2\vec{j} + 3\vec{k}$  and  $\vec{b} = 4\vec{i} + 5\vec{j} + 6\vec{k}$ .

**Answer:**

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2) \vec{i} - (a_1 b_3 - a_3 b_1) \vec{j} + (a_1 b_2 - a_2 b_1) \vec{k}$$

$$\vec{a} \times \vec{b} = (2 \times 6 - 3 \times 5) \vec{i} - (1 \times 6 - 3 \times 4) \vec{j} + (1 \times 5 - 2 \times 4) \vec{k} = -3\vec{i} + 6\vec{j} - 3\vec{k}$$

■ **Example 1.19** Find  $\vec{a} \cdot (\vec{b} + \vec{c})$ , if  $\vec{a} = 5\vec{i} - 4\vec{j}$ ,  $\vec{b} = 7\vec{i} + 8\vec{j}$  and  $\vec{c} = 3\vec{i} - 2\vec{j}$ .

**Answer:**

$$\vec{b} + \vec{c} = 7\vec{i} + 8\vec{j} + 3\vec{i} - 2\vec{j} = 10\vec{i} + 6\vec{j}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = (5\vec{i} - 4\vec{j}) \cdot (10\vec{i} + 6\vec{j}) = 50 - 24 = 26$$

**Exercise 1.18** What is dot product of the vectors,  $3i - 2j + 5k$ , and  $4i + j + 2k$ ?

- A) 16
- B) 20
- C) 26
- D) 30
- E) 32

### 1.8 Vector composition of forces

Given a vector  $\vec{A}$ , we can define its components along the axes as follows. From the tip of the vector, draw lines parallel to the x- and y-axes, and mark the points where these lines intersect the axes. The x- and y-components of  $\vec{A}$  are denoted as  $A_x$  and  $A_y$ , respectively. They can be expressed using trigonometric functions:

$$A_x = A \cos \theta \quad A_y = A \sin \theta \quad (1.19)$$

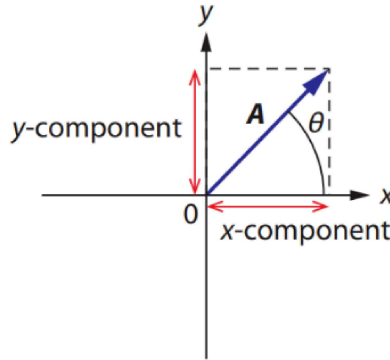


Figure 1.5: Vector composition

where  $A$  is the magnitude of the vector and  $\theta$  is the angle the vector makes with the x-axis. Knowing the components of a vector allows us to determine both the magnitude and the direction of the vector. The magnitude of the vector  $\vec{F}$  can be found using the Pythagorean theorem:

$$F = \sqrt{F_x^2 + F_y^2} \quad (1.20)$$

The direction of the vector, specified by the angle  $\theta$ , can be determined using the tangent function:

$$\theta = \arctan \frac{F_y}{F_x} \quad (1.21)$$

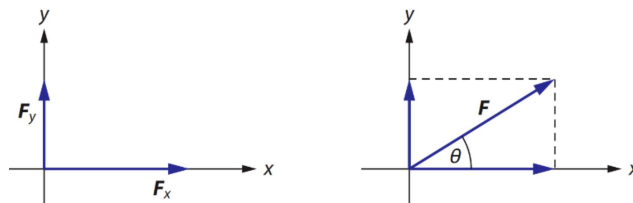


Figure 1.6: Pythagorean theorem

These relationships are fundamental in physics, as they allow for the decomposition and recombination of vectors, which is essential in analyzing forces and motion in multiple dimensions. Understanding how to break down vectors into their components and reconstruct them is a crucial skill in solving many physics problems.

■ **Example 1.20** A vector  $\vec{v}$  has a magnitude of 10 units and makes an angle of  $30^\circ$  with the positive x-axis. Decompose the vector into its x and y components.

**Answer:**

The x-component  $v_x$  is given by  $v_x = v \cos \theta$

$$v_x = 10 \cos 30^\circ = 10 \frac{\sqrt{3}}{2} = 5\sqrt{3}$$

The y-component  $v_y$  is given by  $v_y = v \sin \theta$

$$v_y = 10 \sin 30^\circ = 10/2 = 5$$

Therefore, the vector  $\vec{v}$  can be decomposed into its components as:

$$\vec{v} = 5\sqrt{3} \vec{i} + 5 \vec{j}$$

■

## 1.9 Problems

**Problem 1.1** The fundamental SI units for mass, length, and time, respectively, are

- A) newton, meter, minute.
- B) kilogram, meter, second.
- C) pound, foot, hour.
- D) pound, foot, second.
- E) kilogram, centimeter, hour.

*Source: SAT Subject Test: Physics*

**Problem 1.2** Which of the following is a unit of pressure?

- A)  $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$
- B)  $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
- C)  $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$
- D)  $\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$
- E) none

**Problem 1.3** The energy of a photon of light of frequency  $f$  is given by  $hf$ , where  $h$  is the Planck constant. What are the base units of  $h$ ? (Base units of frequency =  $\text{s}^{-1}$ )

- A)  $\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$
- B)  $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
- C)  $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$
- D)  $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$
- E) none

**Problem 1.4** Which of the following pairs of units are both SI base units?

- A) ampere, degree celsius
- B) ampere, kelvin
- C) coulomb, degree celsius
- D) coulomb, kelvin
- E) none

**Problem 1.5** Which of the following could be measured in the same units as force?

- A) momentum  $\times$  distance
- B) energy / distance
- C) energy  $\times$  distance
- D) energy / time
- E) none

**Problem 1.6** What's is the ratio  $1\mu\text{m}/1\text{Gm}$ ?

- A)  $10^{-3}$
- B)  $10^{-9}$
- C)  $10^{-12}$
- D)  $10^{-15}$
- E)  $10^{15}$

**Problem 1.7** Which quantity has different units from the other three? SI unit of Young modulus is Pascal (Pa).

- A) density  $\times$  volume  $\times$  velocity.
- B) rate of change of momentum.
- C) the Young modulus  $\times$  area.
- D) weight.
- E) the Young modulus.

**Problem 1.8** The unit of work, the joule, may be defined as the work done when the point of application of a force *1newton* is moved a distance of 1 metre in the direction of the force. Express the joule in terms of the base units for mass, length and time, the *kg*, *m* and *s*.

- A)  $\text{kg}.\text{m}^{-1}.\text{s}^2$
- B)  $\text{kg}.\text{m}^2.\text{s}^{-1}$
- C)  $\text{kg}.\text{m}^2.\text{s}^{-2}$
- D)  $\text{kg}.\text{s}^{-2}$
- E)  $\text{kg}.\text{m}^2.\text{s}^{-3}$

**Problem 1.9** A metal sphere of radius  $r$  is dropped into a tank of water. As it sinks at speed  $v$ , it experiences a drag force  $F$  given by  $F = krv$ , where  $k$  is a constant. What are the units of  $k$ ?

- A)  $\text{kg}.\text{m}^2.\text{s}^{-1}$
- B)  $\text{kg}.\text{m}^{-2}.\text{s}^2$
- C)  $\text{kg}.\text{m}^{-1}.\text{s}^{-1}$
- D)  $\text{kg}.\text{m}.\text{s}^{-2}$
- E)  $\text{kg}.\text{m}^{-1}.\text{s}^3$

**Problem 1.10** Decimal sub-multiples and multiples of units are indicated using a prefix to the unit. Which of the following gives the sub-multiples or multiples represented by pico (*p*) and giga (*G*), respectively?

- A)  $10^{-9}$ ,  $10^9$
- B)  $10^{-9}$ ,  $10^{12}$
- C)  $10^{-12}$ ,  $10^9$
- D)  $10^{-12}$ ,  $10^{12}$
- E)  $10^{-9}$ ,  $10^6$

**Problem 1.11** Which expression could be correct for the velocity  $v$  of ocean waves in terms of  $\rho$  the density of seawater,  $g$  the acceleration of free fall,  $h$  the depth of the ocean and  $\lambda$  the wavelength?

- A)  $\sqrt{g \cdot \lambda}$
- B)  $\sqrt{g / h}$
- C)  $\sqrt{\rho \cdot g \cdot h}$
- D)  $\sqrt{g / \rho}$
- E)  $\sqrt{\rho \cdot g \cdot h \cdot \lambda}$

**Problem 1.12** The e.m.f. induced in a coil by a changing magnetic flux is equal to the rate of change of flux with time. Which is a unit for magnetic flux? (Magnetic flux =  $B.A$  and  $F = BIL\sin\theta$ )

- A)  $kg.m^2.s^{-2}.A^{-1}$
- B)  $kg.m^2.s^{-2}.A$
- C)  $kg.m.s^2.A^{-1}$
- D)  $m.s^{-2}.A^{-1}$
- E)  $kg.m^2.s^{-2}.A^{-2}$

**Problem 1.13** The number 300,000,000 can be written as

- A)  $3 \times 10^6$
- B)  $3 \times 10^7$
- C)  $3 \times 10^8$
- D)  $3 \times 10^9$
- E)  $3 \times 10^{-9}$

*Source: SAT Subject Test: Physics*

**Problem 1.14** A millimeter is

- A)  $10^3$  m
- B)  $10^2$  m
- C)  $10^1$  m
- D)  $10^{-3}$  m
- E)  $10^{-6}$  m

*Source: SAT Subject Test: Physics*

**Problem 1.15** How many nanometers are in a kilometer?

- A)  $10^{-9}$
- B)  $10^9$
- C)  $10^{-12}$
- D)  $10^{12}$
- E)  $10^{27}$

*Source: SAT Subject Test: Physics*

**Problem 1.16** The cosine of an angle in a right triangle is equal to the

- A) sine of the angle
- B) tangent of the angle
- C) hypotenuse of the triangle
- D) side adjacent to the angle
- E) ratio of the side adjacent to the angle and the hypotenuse of the triangle

*Source: SAT Subject Test: Physics*

**Problem 1.17** Which of the following quantities is NOT a vector quantity?

- A) displacement
- B) mass
- C) resultant
- D) equilibrant
- E) 10 km at  $30^\circ$  north of east

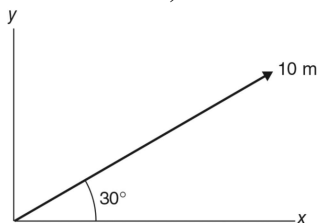
*Source: SAT Subject Test: Physics*

**Problem 1.18** The resultant of the two displacement vectors 3 m east and 4 m north is

- A) 5 m northeast
- B) 7 m northeast
- C) 1 m southwest
- D) 1 m northeast
- E) 12 m northeast

*Source: SAT Subject Test: Physics*

**Problem 1.19** The x-component of the vector shown is most nearly ( $\sin 30^\circ = 0.5$ ,  $\cos 30^\circ = 0.87$ ,  $\tan 30^\circ = 0.58$ )



- A) 10 m
- B) 5 m
- C) 8.7 m
- D) 5.8 m
- E) 100 m

*Source: SAT Subject Test: Physics*

**Problem 1.20** Two displacement vectors, each having a y-component of  $10\text{ km}$ , are added together to form a resultant that forms an angle of  $60^\circ$  from the  $+x$ -axis. What is the magnitude of the resultant? ( $\sin 60^\circ = 0.87$ ,  $\cos 60^\circ = 0.5$ )

- A) 23 km
- B) 40 km
- C) 12 km
- D) 20 km
- E) 30 km

*Source: SAT Subject Test: Physics*

**Problem 1.21** Find the dot product of two vectors having magnitudes of 6 units and 7 units, and the angle between the vectors is  $60^\circ$ .

- A) 6
- B) 7
- C) 42
- D) 21
- E) 0.5

**Problem 1.22** The gradient of a graph of energy (in joules) against displacement (in metres) represents:

- A) momentum
- B) power
- C) acceleration
- D) force
- E) pressure

**Problem 1.23** A physics student wrote the following formulas. Which one is dimensionally consistent?

- A)  $\text{Acceleration} = \text{Force} \times \text{Distance}$
- B)  $\text{Voltage} = \text{Current} + \text{Charge}$
- C)  $\text{Momentum} = \text{Force} \times \text{Velocity}$
- D)  $\text{Energy} = \text{Force} \times \text{Time}$
- E)  $\text{Power} = \text{Work} / \text{Time}$

**Problem 1.24** Which of these has the SI unit of the watt?

- 1. energy / time
  - 2. force  $\times$  velocity
  - 3. mass  $\times$  acceleration  $\times$  velocity
- A) 1 only
  - B) 1 and 2 only
  - C) 2 and 3 only
  - D) 1 and 3 only
  - E) 1, 2 and 3



**Problem 1.25** A chemist measures three nanoparticle diameters: 31000 pm, 0.005  $\mu\text{m}$ , and 4.2 nm. Which sequence orders them from smallest to largest?

- A) 0.005  $\mu\text{m}$ , 4.2 nm, 31000 pm
- B) 4.2 nm, 0.005  $\mu\text{m}$ , 31000 pm
- C) 31000 pm, 4.2 nm, 0.005  $\mu\text{m}$
- D) 0.005  $\mu\text{m}$ , 31000 pm, 4.2 nm
- E) 4.2 nm, 31000 pm, 0.005  $\mu\text{m}$

**Problem 1.26** Which of these physical quantities has the same dimension as work?

- A) pressure
- B) power
- C) momentum
- D) force  $\times$  velocity
- E) pressure  $\times$  volume

**Problem 1.27** Which physical quantity has the dimension  $[ML^0T^{-2}]$ ?

- A) pressure
- B) acceleration
- C) force
- D) energy
- E) mass

**Problem 1.28** Which combination of units is equivalent to the watt?

- A) J·s
- B) N·m/s
- C) N/s
- D) J/s<sup>2</sup>
- E) kg·m<sup>2</sup>/s<sup>2</sup>

**Problem 1.29** The dimensional formula of energy is:

- A)  $[M^1L^2T^{-2}]$
- B)  $[M^0L^1T^{-1}]$
- C)  $[M^1L^1T^{-2}]$
- D)  $[M^1L^2T^{-3}]$
- E)  $[M^0L^0T^0]$

**Problem 1.30** The braking distance  $d$  of a car moving at speed  $v$  is found experimentally to follow the relation:

$$d = \frac{kv^3}{aF}$$

where  $a$  is the cars acceleration magnitude when braking,  $F$  is the frictional force between the tyres and the road, and  $k$  is an experimental constant with unknown dimensions. What are the SI units of  $k$  in this formula?

- A)  $\text{kg}\cdot\text{s}$
- B)  $\text{kg/s}$
- C)  $\text{kg/m}$
- D)  $\text{m/kg}$
- E)  $\text{s/kg}$

**Problem 1.31** The physical dimension of force is:

- A)  $[M^0 L^1 T^{-2}]$
- B)  $[M^1 L^1 T^{-1}]$
- C)  $[M^1 L^1 T^{-2}]$
- D)  $[M^1 L^2 T^{-2}]$
- E)  $[M^1 L^0 T^{-2}]$

**1.10 IMAT Questions**

In this section, all IMAT exam questions related to the measurement chapter from 2011 to the present are provided. By solving these questions, you can familiarize yourself with the exam questions. Additionally, practicing these questions will enhance your problem-solving skills and help you identify key concepts and patterns frequently tested in the IMAT. This thorough preparation is essential for achieving a high score and gaining confidence in tackling the exam.

**IMAT — 2021-2022.** Which of the following expressions gives a quantity that can be measured in joules (J)?

- A) momentum  $\times$  velocity
- B) (charge)<sup>2</sup>  $\times$  resistance
- C) (mass)<sup>2</sup>  $\times$  velocity
- D) specific heat capacity / (mass  $\times$  temperature change)
- E) voltage / (current  $\times$  time)

**IMAT — 2018-2019.** Three spherical particles have the following diameters:  $1650\text{ pm}$ ,  $1.5\text{ nm}$  and  $0.0036\mu\text{m}$ . What is their order of diameter (smallest first)?

- A)  $0.0036\mu\text{m}$ ,  $1.5\text{ nm}$ ,  $1650\text{ pm}$
- B)  $1.5\text{ nm}$ ,  $0.0036\mu\text{m}$ ,  $1650\text{ pm}$
- C)  $1650\text{ pm}$ ,  $1.5\text{ nm}$ ,  $0.0036\mu\text{m}$
- D)  $0.0036\mu\text{m}$ ,  $1650\text{ pm}$ ,  $1.5\text{ nm}$
- E)  $1.5\text{ nm}$ ,  $1650\text{ pm}$ ,  $0.0036\mu\text{m}$

**IMAT — 2016-2017.** Which physical quantity can be measured in joules per metre?

- A) kinetic energy
- B) momentum
- C) power
- D) work
- E) force

**IMAT — 2015-2016.** Which one of the following equations is dimensionally consistent (has consistent units)?

[All the symbols have their usual meanings:  $v$  = velocity;  $F$  = force;  $m$  = mass;  $t$  = time;  $V$  = voltage;  $Q$  = charge;  $R_1, R_2, R_3, R_4$  = resistance]

- A)  $\text{energy} = (\frac{1}{2}mv^2) + F.v$
- B)  $\text{resistance} = R_1 + R_2 + (1/R_3) + (1/R_4)$
- C)  $\text{temperature change} = \text{energy} \times m \times \text{specific heat capacity}$
- D)  $\text{acceleration} = (\frac{1}{2}vt^2) + (F/m)$
- E)  $\text{electrical current} = (V/R_1) + (Q/t)$

**IMAT — 2013-2014.** In the expressions below:

$g$  = gravitational acceleration;  $h$  = height;  $m$  = mass;  $R$  = electrical resistance;  $t$  = time;  $v$  = velocity;  $V$  = voltage.

Which of the following expressions have units of power?

1.  $\frac{mv^2}{2t}$     2.  $\frac{V^2}{R}$     3.  $\frac{mgh}{t}$

- A) 1 and 2 only
- B) 1 only
- C) 2 and 3 only
- D) 1 and 3 only
- E) 1,2 and 3

**IMAT — 2011-2012.** The law of gravitation states that the gravitational force between two bodies of mass  $m_1$  and  $m_2$  is given by:

$F = Gm_1m_2/r^2$ . What is the gravitational force between the Earth and the Moon?

$$G = 7 \times 10^{-11} N.m^2.kg^{-2}, \quad m_1 = 6 \times 10^{24} kg, \quad m_2 = 7 \times 10^{22} kg, \quad r = 4 \times 10^8 m$$

- A) 1 and 2 only
- B) 1 only
- C) 2 and 3 only
- D) 1 and 3 only
- E) 1,2 and 3